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Weinberg Angle and Pion Beta Decay in the Spinor Strong Interaction Theory

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From an isospin $SU(2) \times$ hypercharge $U(1)$ gauge-invariant meson action for K^+, $\pi^+ \to \mu^+\nu$, a prediction of the Cabbibo angle was obtained earlier. Using another result of the spinor strong interaction theory that the s quark is only a little heavier than the d quark, the above action can be extended to a weakly broken $SU(3)$ one. The Weinberg angle is found to be 30 deg in the limit of this $SU(3)$ symmetry. In the approximation and context entailing this limit, the Weinberg angle can be removed from the list of undetermined fundamental parameters in electroweak theory. The spinor strong interaction Lagrangian used above and applied to the decay $\pi^+ \to e^+\nu$ is shown to hold for $\pi^+ \to \pi^0 e^+\nu$ as well, rendering the conserved vector current hypothesis hitherto required to account for the latter decay superfluous.

1. INTRODUCTION

The current approach to low-energy meson theory is phenomenological. The Cabbibo and Weinberg angles are considered as fundamental parameters to be determined by data. Two different Lagrangians, one of them including the conserved vector current hypothesis, hereafter abbreviated by CVC, are required to account for the closely related decays $\pi^+ \to e^+\nu$ and $\pi^+ \to \pi^0 e^+\nu$. Further, quark confinement, the apparent absence of ground-state scalar mesons, the $U(1)$ problem, and the nonobservation of the Higgs boson remain as basic unsolved problems. These problems seem to be unrelated in the current view and have persisted for decades (Lee, 1981; Kallén, 1964). Their resolution is impeded by the lack of a nonphenomenological strong interaction theory at low energies.

Such a theory was proposed a few years ago (Hoh, 1993) and developed recently (Hoh, 1994, 1996). Both quark coordinates x_{I} and x_{II} are transformed

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into an observable laboratory coordinate X and a nonobservable relative coordinate $x = x_{\mathrm{II}} - x_{\mathrm{I}}$, which is thus a hidden variable. Hadron wave function amplitudes are finite, which is required for quark confinement and for the generation of the masses of the gauge bosons.

This is in contrast with the conventional amplitude $|\psi| = 1/\sqrt{2EV}$ for a Klein–Gordon hadron, where E is its energy and V the volume of the normalization box. This amplitude vanishes as $V \to \infty$ and hence cannot generate the mass of the gauge boson. Actually, the normalization $\int d^3\mathbf{X}|\psi|^2 = 1$, which leads to the above amplitude, is a choice by convention and is not any physical requirement. This choice is abandoned in the spinor strong interaction theory (Hoh, 1993, 1994, 1996, 1997).

Based upon this theory, the problems mentioned in the first paragraph, except for two, have been basically resolved in a unified manner. This paper is a further development of recent work (Hoh, 1997), hereafter denoted by I, on $\pi, K \to e, \mu + \nu$. It resolves the two remaining problems, namely the origin of the Weinberg angle and the relationship of the above two modes of π^+ decay.

In I, it was shown that the Cabbibo angle ϑ_C is determinable from the K and π masses and is not a fundamental parameter in that context. The gauge boson mass squared M_w^2 is proportional to the slope of the confinement potential and the scale of the relative time between the quarks, whereby the K's or the π's play the role of Higgs bosons. The successful results of purely leptonic interactions of the standard model can be taken over without the aid of the unseen Higgs bosons.

In this paper, the notations and structure of I will be followed, unless stated otherwise. $SU(2) \times U(1)$ gauge invariance for the meson isodoublets in I is generalized to a weakly broken $SU(3)$ one. In Section 2, such an $SU(3)$ meson action is justified. $SU(3)$ gauge invariance is shown in Section 3, as is invariance with respect to inclusion of internal functions. The total action for the meson, gauge boson, and lepton system is given in Section 4. In Section 5, the $SU(2)_I \times U(1)_Y$ gauge-invariant part of the $SU(3)$ meson action is separated out for coupling to the leptons. The origin of the Weinberg angle is exposed. In Section 6, the action for $\pi^+ \to e^+\nu$ in I is reproduced for application to $\pi^+ \to \pi^0 e^+\nu$. The first- and second-order perturbational treatment of this action leading to the amplitude for the last decay is worked out in Sections 7 and 8. In Section 9, it is shown that this amplitude and the decay rate are nearly the same as those in the literature, given in the Appendix, based on a phenomenological Lagrangian and CVC.

2. *SU*(3) MESON ACTION

In I, $K^+ \to \mu^+\nu$ was considered as a typical case. As was mentioned in Section 11 of I, the decay rate formula for this process also holds for $D^+ \to \mu^+\nu$ with the D mass replacing the K mass.

In the basic meson equation (4.11) of Hoh (1993) or the meson action (2.1a) below, flavor dependence enters S_{MT} only through the term $M_m^2 = (m_a + m_b)^2/4$, where a and b are the quark flavors. Application of the spinor strong interaction theory to meson spectra (Hoh, 1996) led to the quark masses $m_u = 0.6592$ Gev, $m_d - m_u = 0.00215$, $m_s - m_d = 0.08175$, and $m_c = 1.6215$. Thus, the s quark is only 11% heavier than the d quark and M_m^2 for K and π differ by 12.8%. The large difference between K and π masses arises from subtraction of two nearly equal but large numbers. Similarly, M_m^2 for $D^-(d\bar{c})$ and $D_s^-(s\bar{c})$ differ only by 7.3%.

These results allow that the $SU(2)_I \times U(1)_Y$ gauge-invariant action of (I 2.4) can be approximately generalized to a $SU(3)$ one. Here, I denotes isospin and Y hypercharge. The form of such an action is the same as (I 2.4),

$$\begin{aligned} S_{MT} &= \int d^4X\, \mathscr{L}_{MT} \\ &= -\int d^4X \int d^4x\, \frac{1}{2} \\ &\quad \times \left\{ \left(\frac{1}{2} \{[(1-a)D^{\dot{b}a} - \partial_x^{\dot{b}a}]\Psi_T^\dagger(X)\chi_a^{\dot{e}}(x)\}\{(aD_{f\dot{e}} + \partial_{xf\dot{e}})\Psi_T(X)\chi_{\dot{b}}^f(x)\} \right.\right. \\ &\quad + \frac{1}{2}\{(aD^{\dot{b}a} + \partial_x^{\dot{b}a})\Psi_T^\dagger(X)\chi_a^{\dot{e}}(x)\} \\ &\quad \left. \times \{[(1-a)D_{f\dot{e}} - \partial_{xf\dot{e}}]\Psi_T(X)\chi_{\dot{b}}^f(x)\} + \text{h.c.} \right) \\ &\quad \left. + (\phi_p - M_m^2)\Psi_T^\dagger(X)\Psi_T(X)[\psi_{\dot{b}}^{\dot{c}}(x)\chi_{\dot{c}}^{b}(x) + \text{c.c.}] \right\} \end{aligned} \tag{2.1a}$$

Here, however,

$$\Psi_T(X) = \begin{pmatrix} \Psi_{T1}(X) \\ \Psi_{T2}(X) \\ \Psi_{T3}(X) \end{pmatrix} \tag{2.1b}$$

are associated with the triplet D^0 $(u\bar{c})$, D^-, and D_s^- or the triplet K^+, K^0, and a virtual $\eta_s(s\bar{s})$. As is shown in Section 5 of Hoh (1994), η_s is forbidden by $U(1)$ gauge invariance. Further,

$$D^{a\dot{b}} = \partial^{a\dot{b}} + \frac{i}{2} g \sum_{l=1}^{8} \lambda_l W_l^{a\dot{b}} \tag{2.1c}$$

where λ_l denotes the Gell-Mann matrices and W_l are eight real gauge fields. This repalces (I 2.3d):

$$D_2^{a\bar{b}} = \partial^{a\bar{b}} + \frac{i}{2} g \,\sigma\, \mathbf{W}^{a\bar{b}} + \frac{i}{2} g' Y C^{a\bar{b}} \tag{2.2}$$

where σ are the Pauli matrices and $\boldsymbol{W} = (W_1, W_2, W_3)$.

Note that a generalization of the $SU(3)$ gauge fields in (2.1c) to $SU(3) \times U(1)_C$, where C denotes the charm quantum number, implies an approximate $SU(4)$ symmetry. There is, however, no such approximative symmetry, because $m_c >> m_s$. Therefore, such a $U(1)_C$ group is excluded.

Furthermore, such a generalization to (2.1b) and (2.1c) is not possible in the conventional phenomenological approach (Lee, 1981; Kallén, 1964) due to (i) the fact that the s quark is generally appreciably heavier than the d quark and (ii) the fact that currents, which correspond to the Ψ's here, involving an s quark are suppressed by a factor of $\sin \vartheta_C$ in the phenomenological Lagrangians of the Cabbibo theory.

3. $SU(3)$ GAUGE INVARIANCE AND INCLUSION OF INTERNAL FUNCTIONS

Gauge transformations of (2.1) follow (Ludwig and Falter, 1988). Neglecting the quark mass differences so that M_m^2 is the same for all three members of (2.1b), we find that (2.1) is invariant under

$$\Psi_T \to \Psi_T' = P_T \Psi_T \tag{3.1a}$$

$$P_T = \exp\left[\frac{i}{2} g \sum_{l=1}^{8} \lambda_l \varphi_l (X)\right] \tag{3.1b}$$

$$\lambda_l W_l^{a\bar{b}} \to P_T(\lambda_l W_l^{a\bar{b}}) P_T^{-1} + \frac{i}{g} (\partial^{a\bar{b}} P_T) P_T^{-1} \tag{3.1c}$$

where φ_l are real phases. In addition, following Section 8 of I, we make the generalizations below to include internal functions for the D triplet:

$$\Psi_T \to \Psi_T \,\xi_T = \begin{pmatrix} \Psi_{T1}\xi_{T1} \\ \Psi_{T2}\xi_{T2} \\ \Psi_{T3}\xi_{T3} \end{pmatrix} \tag{3.2a}$$

where

$$\xi_{T1} = (z^1 u_4 - u^1 z_4)/\sqrt{2}, \qquad \xi_{T2} = (z^2 u_4 - u^2 z_4)/\sqrt{2}$$
$$\xi_{T3} = (z^3 u_4 - u^3 z_4)/\sqrt{2} \tag{3.2b}$$

If the subscript 4 is replaced by 3, (3.2) represents the triplet K^+, K^0, and η_s.

Dropping the spinor indices, (I 8.5) is generalized to

$$W_1 \mp iW_2 = \sqrt{2}\, W^{\pm} \to \sqrt{2}\, W_{\mathrm{I}}^{\pm}\, D_{I\pm}$$
$$W_4 \mp iW_5 = \sqrt{2}\, W_V^{\pm} \to \sqrt{2}\, W_V^{\pm}\, D_{V\pm}$$
$$W_6 \mp iW_7 = \sqrt{2}\, W_U^{\pm} \to \sqrt{2}\, W_U^{\pm}\, D_{U\pm} \tag{3.3}$$

$$D_{I+} = z^1\, \partial/\partial z^2 - z_2\, \partial/\partial z_1 + z \to u, \qquad D_{I-} = -D_{I+}^*$$
$$D_{V+} = z^1\, \partial/\partial z^3 - z_3\, \partial/\partial z_1 + z \to u, \qquad D_{V-} = -D_{V+}^*$$
$$D_{U+} = z^2\, \partial/\partial z^3 - z_3\, \partial/\partial z_2 + z \to u, \qquad D_{U-} = -D_{U+}^* \tag{3.4}$$

Operating on (3.2b) by (3.4) leads to the results

$$D_{I+}\, \xi_{T2} = \xi_{T1}, \qquad D_{I-}\, \xi_{T1} = \xi_{T2}, \qquad D_{U+}\, \xi_{T3} = \xi_{T2}, \qquad D_{U-}\, \xi_{T2} = \xi_{T3}$$
$$D_{V+}\, \xi_{T3} = \xi_{T1}, \qquad D_{V-}\, \xi_{T1} = \xi_{T3} \tag{3.5}$$

with all other $D\xi_T$ combinations vanishing.

As in Section 8 of I, (3.2)–(3.4) are substituted into (2.1). Making use of (3.5) and (I 8.3),

$$z^k z_r = u^k u_r = \delta_r^k, \qquad z^k u_r = 0 \tag{3.6}$$

it is seen that (2.1) is invariant under these substitutions.

4. THE TOTAL ACTION

The action (I 2.1) employed for $K^+ \to \mu^+\nu$ is generalized to

$$S_{ML} = S_{MT} + S_{F8} + S_{Lr} + S_{Ll} + S_{Lm} \tag{4.1}$$

Here, S_{F8} is a generalization of the $SU(2) \times U(1)$ gauge-invariant boson action (I 2.2),

$$S_{F8} = -\frac{1}{4}\int d^4 X \sum_{l=1}^{8} G_l^{\mu\kappa}\, G_{l\mu\kappa} \tag{4.2a}$$

$$G_l^{\mu\kappa} = \partial^\mu W_l^\kappa - \partial^\kappa W_l^\mu - f_{jkl} g W_j^\mu W_k^\kappa \tag{4.2b}$$

$$[\lambda_j, \lambda_k] = 2if_{jkl}\lambda_l \tag{4.2c}$$

where μ and κ refer to the components of a conventional four-vector, which are employed in place of the spinor indices a, b in (3.1c). f_{jkl} are the $SU(3)$ structure constants. S_{F8} is invariant under the $SU(3)$ gauge transformations (3.1b) and (3.1c). Analogous to Section 2 of I, the massless right-handed singlet and left-handed doublet lepton actions are

$$S_{Lr} = -\frac{i}{4}\int d^4\, X\, \chi_{\mu a}(\partial^{a\bar{b}} + \frac{i}{2} Yg'\, C^{a\bar{b}})\chi_{\mu\bar{b}} + \text{c.c.} \tag{4.3a}$$

$$S_{Ll} = -\frac{i}{4}\int d^4\, X\, \psi_d^{\dot{a}T}\, D_{2\dot{a}b}\, \psi_d^b + \text{c.c.} \tag{4.3b}$$

$$\psi_d^b = \begin{pmatrix} \psi_\nu^b \\ \psi_\mu^b \end{pmatrix} \tag{4.3c}$$

where T denotes transpose, μ refers to the muon, and ν to the neutrino.

Note that ψ and χ are interchanged in (I 2.3a)–(I 2.3c), which is an error inconsistent with the definitions (A3) of Hoh (1993), but turns out not to affect the results in I. The lepton mass term S_{Lm} is the same as (I 2.3c). The gauge invariance properties of the lepton actions are the same as those discussed above (I 2.6).

5. ORIGIN OF THE WEINBERG ANGLE

The action S_{ML} of (4.1) is not self-consistent in that there are eight gauge fields in S_{MT} and S_{F8}, but only four in S_L. Use (3.3), drop the spinor indices, and write out the sum in (2.1c) in the form

$$\sum_{l=1}^{8} \lambda_l W_l = \left(\begin{array}{cc|c} W_3 + \frac{1}{\sqrt{3}} W_8 & \sqrt{2}W_I^+ & \sqrt{2}W_V^+ \\ \sqrt{2}W_I^- & -W_3 + \frac{1}{\sqrt{3}} W_8 & \sqrt{2}W_U^+ \\ \hline \sqrt{2}W_V^- & -\sqrt{2}W_U^- & -\frac{2}{\sqrt{3}} W_8 \end{array}\right) \tag{5.1}$$

where the $SU(2)_I \times U(1)_Y$ subgroup gauge fields are confined by the dashed lines. These fields together with the first two of (2.1b) can now be associated with the corresponding ones in (4.3).

In I, (2.2) operates on the first two components in (2.1b) representing D^0 and D^- or K^+ and K^0, which have $Y = 1$. Compare now (2.2) to (2.1c) and (5.1), limited to $SU(2) \times U(1)$ gauge fields separated off by the dashed lines. Identifying $C^{a\bar{b}}$ with $W_8^{a\bar{b}}$, we find that the Weinberg angle ϑ_w is given by

$$\tan \vartheta_{\mathrm{w}} = g'/g = 1/\sqrt{3}, \qquad \sin^2\vartheta_w = 0.25, \qquad g = 2e \tag{5.2}$$

in the limit of $SU(3)$ symmetry. Thus, the Weinberg angle comes mainly from the $1/\sqrt{3}$ factor in λ_8.

Data (Particle Data Group, 1996) give an average $\sin^2\vartheta_{\mathrm{w}} = 0.2241$, which leads to $g'/g = (1/\sqrt{3})/1.0743$. This descrepancy of 7.43% may be attributed to the $SU(3)$ symmetry-breaking effect due to the unequal s and d quark masses. It is expected to be of the same order of magnitude as the differences in M_m^2 for the different members of the triplet (3.2a). Indeed, this appears to be the case since the last differences are 7.3% and 12.8% for the D triplet and K^+, K^0, and η_s, respectively, as was shown above (2.1).

In the limit of $SU(3)$ symmetry, the minimal substitutions invariant under $U(1)$ and $SU(2)$ gauge transformations now share in an appealing manner the same coupling constant e:

$$\partial \to \partial + ieA \qquad U(1) \tag{5.3a}$$

$$\partial \to \partial + ie\sigma \mathrm{W} \qquad SU(2) \tag{5.3b}$$

where (2.2) and (5.2) have been consulted. To the approximation entailing this limit, the Weinberg angle can thus be removed from the list of fundamental parameters to be determined by other data in the standard electroweak model.

In (I 11.4), it was shown that the Cabbibo angle was derivable from the K and π masses. The difference of these masses arises from the relatively small difference in the s-quark and d-quark masses (Hoh, 1996). Whether this small mass difference, which breaks the $SU(3)$ symmetry above, is also the universal origin of the Cabbibo angle has not been shown in other contexts, such as the weak decay of baryons. Should this turn out to be the case, the Cabbibo angle can also be removed from the list of unknown fundamental parameters.

6. ACTION FOR PION BETA DECAY $\pi^+ \to \pi^0 e^+\nu$

The action for the pion weak decay $\pi^+ \to e^+\nu$ is given by (I 9.1):

$$S_{3ML} = S_{M3} + S_{F3} + S_{Lr} + S_{Ll} + S_{Lm} \tag{6.1}$$

Here, S_{F3} is the same as (4.2a) with $l = 1$, 2, and 3 and S_{M3} is (2.1a) with

$$\Psi_t(X) = \begin{pmatrix} \Psi_{t+}(X) \\ \Psi_{t0}(X) \\ \Psi_{t-}(X) \end{pmatrix} \tag{6.2a}$$

$$D^{a\bar{b}} = \partial^{a\bar{b}} + \frac{i}{2} g \begin{pmatrix} \underline{W}_3 & \sqrt{2}W_I^+ & \underline{0} \\ \sqrt{2}W_I^- & \underline{0} & \sqrt{2}W_I^+ \\ 0 & \sqrt{2}W_I^- & -W_3 \end{pmatrix}^{a\bar{b}} \tag{6.2b}$$

The subscripts $\pm$ and 0 refer to $\pi^{\pm}$ and π^0, respectively.

$SO(3)$ gauge invariance of S_{MT} with (6.2) as well as invariance with respect to inclusion of internal functions have been shown in Section 7 of Hoh (1994) and Section 9 of I, respectively. In the following, it will be shown that (6.1) also holds for $\pi^+ \to \pi^0 e^+ \nu$, apart from some redefinitions involving multiplications by $2\sqrt{2}$.

The last decay is similar to the decay $D^+ \to D\gamma$ (Hoh, n.d.; hereafter denoted by II), in that the final hadron state is not vacuum, as is the case for $\pi^+ \to e^+\nu$, but another hadron. Therefore, the quantization procedure of II will be adopted.

7. FIRST-ORDER PERTURBATION AND DECAY AMPLITUDE

The action (6.1) is split up into zeroth-, first-, and second-order parts according to (I 2.6), with S_{M2} and S_{F2} replaced by S_{M3} and S_{F3}. The zeroth-order terms account for a π^+ at rest. The meson wave functions in (2.1a) and (6.2a) are written in the form of (II 3.1),

$$\Psi_t(X)\Psi^{ab}(x) = \Sigma_{\mathbf{K}}\, b_{\mathbf{K}} \exp[-iE_{\mathbf{K}}X^0 + i\mathbf{KX} - (x^0/\tau_0)^2]$$

$$\times [\psi_{\mathbf{K}}(\mathbf{x})\delta^{ab} - \psi_{\mathbf{K}}(\mathbf{x})\sigma^{ab}]$$

$$b_{\mathbf{K}} = a_{\mathbf{K}} + a_{\mathbf{K}}^{(1)}(X^0), \qquad \psi \to \chi \tag{7.1}$$

where the relative time scale τ_0 factor is included, as in (I 3.2). $E_{\mathbf{K}}$ is the energy of the meson and $a_{\mathbf{K}}$ the annihilation operator for it. $a_{\mathbf{K}}^{(1)}(X^0)$ is the associated first-order decay amplitude varying slowly with the time X^0. Summation over $\mathbf{K}$ is symbolic because only one $\mathbf{K}$ value is allowed. This is due to the nonlinear nature of the meson equations (4.11) and (4.12) of Hoh (1993).

Analogous to (I 4.3b, c), let

$$|i\rangle = |\pi^+(\mathbf{K}_+ = 0)\rangle, \qquad \langle f| = \langle \pi^0\,(\mathbf{K}_0)|$$

$$|a_0|i\rangle = |0\rangle, \qquad \langle f|a_{0\mathbf{K}}^* = \langle 0| \tag{7.2}$$

Here, the subscripts + and 0 denote π^+ and π^0, respectively. Thus, E_{+0} below is the energy of a π^+ at rest and $E_{0\mathbf{K}}$ is the energy of a π^0 with momentum $\mathbf{K}_0$. Let the first-order terms of S_{3ML} be sandwiched between $\langle f|$ and $|i\rangle$. Collecting terms containing $a_{+0}^{(1)}\,(X^0)$, as in (I 4.4) or (II 3.5), leads to

$$-\frac{i}{2}\, E_{+0}\, S_{fi} V_M \int d^4x\, \psi_{+0}^2\,(\mathbf{x}), \qquad V_M = \int d^3\mathbf{X}$$

$$S_{fi} = \langle f|a_{+0}^{(1)*}\,(X^0 \to \infty)\, a_{+0}|i\rangle \tag{7.3}$$

Terms linear in W^- are basically (I 4.6), modified to include the final π^0 in motion, and read

$$-\frac{\sqrt{2}}{4}g\int d^4X\int d^4x\, e^{i(E_{0K}-E_{+0})X^0-i\mathbf{K}_0\mathbf{X}}\Big\{(E_{+0}+E_{0\mathbf{K}})\,\chi_{0\mathbf{K}}\chi_{+0}\,W_0^-$$

$$-\mathbf{K}_o\mathbf{W}^-\,\chi_{0\mathbf{K}}\chi_{+0}-\frac{1}{2}(E_{+0}+E_{0\mathbf{K}})\,\mathbf{W}^-\,(\chi_{0\mathbf{K}}-\psi_{0\mathbf{K}})$$

$$+(\mathbf{W}^-\,\chi(\chi_{0\mathbf{K}}+\psi_{0\mathbf{K}}))\frac{\partial}{\partial\mathbf{x}}\chi_{+0}-\chi_{+0}\,\mathbf{W}^-\left(\frac{\partial}{\partial\mathbf{x}}\chi(\chi_{0\mathbf{K}}+\psi_{0\mathbf{K}})\right)\Big\}$$

$$W^{-a\bar{b}}=W_0^-\delta^{a\bar{b}}-\sigma^{a\bar{b}}\mathbf{W}^- \tag{7.4}$$

Due to the small difference in the π^+ and π^0 masses, the momentum $\mathbf{K}_0$ is small, just as the momentum of D is small in $D^*\to D\gamma$ of II. Thus, the treatment of slowly moving pseudoscalar mesons in Appendix B of II applies here also. It is based upon expansion in another small parameter $\varepsilon_0=\mathbf{K}_0/E_{0\mathbf{K}}$ of (II B5). To first order in ε_0, (II B7) and (II B8) show that $-\psi_{+0}=\chi_{+0}=\chi_{0\mathbf{K}}=-\psi_{0\mathbf{K}}$ and that $\chi_{0\mathbf{K}}$ and $\psi_{0\mathbf{K}}$ are of order ε_0. Using this result and equating the first of (7.3) to the negative of the first of (7.4) yields a decay amplitude corresponding to (I 4.7),

$$S_{Fi}=\frac{i}{\sqrt{2}}\frac{g}{V_M}\int d^4X\,e^{i(E_{0\mathbf{K}}-E_{+0})X^0-i\mathbf{K}_0\mathbf{X}}\frac{1}{E_{+0}}$$

$$\times\,((E_{+0}+E_{0\mathbf{K}})W_0^-+\mathbf{K}_0\mathbf{W}^-+\Delta_{IX}) \tag{7.5a}$$

$$\Delta_{IX}=-2\mathbf{K}_0\mathbf{W}^-+\Big\{\int d^4x\Big[-\frac{1}{2}(E_{+0}+E_{0\mathbf{K}})\,\mathbf{W}^-\,(\chi_{0\mathbf{K}}-\psi_{0\mathbf{K}})$$

$$+(\mathbf{W}^-\times(\chi_{0\mathbf{K}}+\psi_{0\mathbf{K}}))\frac{\partial}{\partial\mathbf{x}}\chi_{+0}$$

$$-\chi_{+0}\,\mathbf{W}^-\left(\frac{\partial}{\partial\mathbf{x}}\times(\chi_{0\mathbf{K}}+\psi_{0\mathbf{K}})\right)\Big]\Big\}\Big/\int d^4x\,\psi_{+0}^2(\mathbf{x}) \tag{7.5b}$$

8. SECOND-ORDER PERTURBATION AND GAUGE BOSON MASS

This section corresponds to Section 5 of I. Variation of (4.1) with respect to $W^{+a\bar{b}}$ and collection of the second-order terms leads to an expression analogous to (I 5.1),

$$\frac{1}{2}\Box W^{-}_{a\dot{b}} - \frac{1}{4}\partial_{a\dot{b}}(\partial^{d\dot{c}}\, W^{-}_{\dot{c}d}) + g^{2}(\ldots) + \frac{1}{2\sqrt{2}}\, g\, \psi_{ea}(X)\psi_{\nu\dot{b}}(X)$$
$$+ \frac{1}{32}\, g^{2}\int d^{4}x\, W^{-}_{\dot{e}f}[\Psi^{*}_{\dot{t}+}\chi^{\dot{e}}_{+a}\Psi_{t+}\chi^{f}_{+\dot{b}} + \chi \to \psi + \text{c.c.}] = 0 \quad (8.1)$$

where the subscript e refers to the positron. The $g^{2}(\ldots)$ term denotes W-cubed terms. The first three terms are dropped because they are much smaller than the last term, associated with the gauge boson mass. Making use of (7.1) with $b_{\mathbf{K}} = 1$ and the second of (7.4), we find that (8.1) leads to

$$W^{-}_{0} = \frac{g}{2\sqrt{2}\, M_{w}^{2}}(\psi_{e1}\psi_{\nu 1} + \psi_{e2}\psi_{\nu 2}) \tag{8.2a}$$

$$\mathbf{W}^{-} = \frac{g}{2\sqrt{2}\, M_{w}^{2}}(-\sigma^{a\dot{b}}\psi_{ea}\psi_{\nu\dot{b}}) \tag{8.2b}$$

$$M_{w}^{2} = \frac{1}{4}\, g^{2}\int d^{3}\mathbf{x}\, \chi^{2}_{+0}(x)\int dx^{0} e^{-2(x^{0}/\tau_{0})^{2}}$$
$$= \pi\sqrt{2\pi}\, g^{2}\beta_{m0}\tau_{o} = (80.33\ \text{GeV})^{2} \tag{8.2c}$$

where (8.2c) is the same as (I 5.2b) and β_{m0} is the slope of the confinement potential (7.6b) of Hoh (1993).

The plane-wave functions for free leptons entering (8.2) satisfy the Dirac equation in spinor form [see, e.g., (4.5) of Hoh (1993), putting $V_{PB} = 0$ there]. As in the literature (Kallén, 1964), negative- energy solutions, to be denoted by the superscript $(-)$, with negative momentum are assigned to the positron, and the complex conjugate of the positive-energy solutions, to be denoted by the superscript $(+)$, with positive momentum are assigned to the neutrino. These read

$$\psi_{\nu\dot{b}} = V_{\nu}^{-1/2}\, e^{i(E_{\nu}X^{0} - \mathbf{K}_{\nu}\mathbf{X})}\, u^{(+)}_{\nu\dot{b}}(\mathbf{K}_{\nu})$$
$$\psi_{ea} = V_{e}^{-1/2}\, e^{i(E_{e}X^{0} - \mathbf{K}_{e}\mathbf{X})}\, u^{(-)}_{ea}(-\mathbf{K}_{e}) \tag{8.3a}$$

$$u^{(+)}_{\nu\dot{1}}(\mathbf{K}_{\nu}) = (2E_{\nu})^{-1/2}(K_{\nu 1} - i\, K_{\nu 2},\ - E_{\nu} - K_{\nu 3})$$
$$u^{(+)}_{\nu\dot{2}}(\mathbf{K}_{\nu}) = (2E_{\nu})^{-1/2}(E_{\nu} - K_{\nu 3},\ - K_{\nu 1} - iK_{\nu 2})$$
$$u^{(-)}_{e1}(-\mathbf{K}_{e}) = [2(E_{e} + m_{e})]^{-1/2}(-K_{e1} - iK_{e2},\ E_{e} + m_{e} + K_{e3})$$
$$u^{(-)}_{e2}(-\mathbf{K}_{e}) = [2(E_{e} + m_{e})]^{-1/2}(-E_{e} - m_{e} + K_{e3},\ K_{e1} - iK_{e2}) \tag{8.3b}$$

where m_{e} is the positron mass. The commas in the parentheses separate the spin-up and -down cases.

Inserting (8.2) into (7.5) and making use of (8.3a) leads to

$$S_{fi} = i\sqrt{2}\,(V_\nu V_e)^{-1/2}\, V_M^{-1} G E_{+0}^{-1} \int d^4 X$$
$$\times \{\exp[i(E_{0\mathbf{K}} + E_e + E_\nu - E_{+0})X^0$$
$$- i(\mathbf{K}_0 + \mathbf{K}_e + \mathbf{K}_\nu)\mathbf{X}]\}\,(1 + \Delta_I)$$
$$\times \{(E_{+0} + E_{0\mathbf{K}})\, u_{ea}^{(-)}\,(-\mathbf{K}_e)\, u_{\nu\dot{a}}^{(+)}\,(\mathbf{K}_\nu)$$
$$- \mathbf{K}_0\, \sigma^{a\dot{b}}\, u_{ea}^{(-)}\,(-\mathbf{K}_e)\, u_{\nu\dot{b}}^{(+)}\,(\mathbf{K}_\nu)\} \tag{8.4a}$$

$$\Delta_I = \Delta_{IX}/(W_0^-(E_{+0} + E_{0\mathbf{K}}) + \mathbf{K}_0\mathbf{W}^-) \tag{8.4b}$$

9. DECAY RATE

In the Appendix, the decay amplitude $S_{fi(K)}$ and decay rate $1/\tau$ based upon CVC (Kallén, 1964) are given. Compare this amplitude to S_{fi} of (8.4a). With (A2–5, 6, 24) of Kallén (1964) and (8.3b), it can be shown that the expressions in braces in (8.4a) and (A1) are equal, noting that $E_{0\mathbf{K}} = E_{00}$ to first order in ε_0 according to (II B6c) and (II B8a). Dropping the Δ_I term in (8.4a) for the moment, the comparison yields

$$|S_{fi}| = \sqrt{2}\; 2\,(E_{0\mathbf{K}}/E_{+0})^{1/2} |S_{fi(\mathbf{K})}| \tag{9.1}$$

The factor $2\,(E_{0\mathbf{K}}/E_{+0})^{1/2}$ stems from the following. In the Appendix, the amplitudes of the wave functions of π^+ and π^0 are $(2E_{+0}\,V_M)^{-1/2}$ and $(2E_{0\mathbf{K}}\,V_M)^{-1/2}$, respectively. There is no such normalized amplitude in the spinor strong interaction theory, as was pointed out in the introduction and is seen in (7.1). The factor $\sqrt{2}$ comes from the definition of W^- in (3.3) and is seen in (6.2b).

The factor $2\sqrt{2}$ can be absorbed into M_W^2. Divide (8.1) by $2\sqrt{2}$ and, instead of (8.2c), let

$$M_W^2 = \frac{1}{2\sqrt{2}}\frac{1}{4}\, g^2 \int d^3\mathbf{x}\, \chi_{+0}^2(x) \int dx^0 e^{-2(x^0/\tau_0)^2}$$
$$= \frac{1}{2}\,\pi\sqrt{\pi}\, g^2\, \beta_{m0}\, \tau_0 = (80.33\ \text{GeV})^2 \tag{9.2}$$

This leads to $W^- = (g/8M_w^2)(\ \ldots\ )$ instead of (8.2).

Consequently, S_{fi} of (8.4a) $\rightarrow S_{fi}/2\sqrt{2}$, and (9.1) goes over to

$$|S_{fi}| = (E_{0\mathbf{K}}/E_{+0})^{1/2}\ |S_{fi(K)}| \tag{9.3}$$

With the data in Particle Data Group (1996), (A4) yields the decay time $\tau = 2.457$ sec, which is just outside the error limit of the measured 2.54 sec $\pm 3.3\%$ (Particle Data Group, 1996). Replacing $S_{fi(K)}$ in (A2) by S_{fi} in (9.3), the decay time is increased by the factor E_{+0}/E_{00} and becomes $\tau = 2.54$ sec, in agreement with the measured value.

This agreement is obtained by neglecting the Δ_I term in (8.4a), which is of order ε_0. This can be seen from (8.4b) and (7.5b), where W_0^- and $\mathbf{W}^-$ are of the same order. The energy Δ of (A3a) less m_e is available to impart the three decay products with momenta. The upper limit of the π^0 momentum $|\mathbf{K}_0|$ is obtained when both the positron and the neutrino move in the same direction. Let the average value of $|\mathbf{K}_0|$ be half this limit; $\varepsilon_0 \lesssim 1.49\%$ is obtained. The decay time τ is proportional to $(1 + \Delta_r)^{-2}$, so that the predicted $\tau = 2.54$ sec above can be off by about 3%.

In conclusion, the spinor strong interaction Lagrangian (6.1) accounts for both $\pi^+ \to e^+\nu$ and $\pi^+ \to \pi^0 e^+\nu$, without recourse to CVC relied upon in the Appendix. This is achieved by redefining M_W^2 according to (9.2), which replaces (8.2c) or (I 5.2b).

This redefinition also leads to the result that the waek decay rate $\Gamma(K$ or $\pi \to \mu$ or $e + \nu)$ of (I 6.5) and (I 10.4) should be reduced by a factor of 8. This does not alter the main results of I, but changes some of the derived parameters in Section 11 of I. The altered values are $\Omega/L_M^6 = (0.207\ \text{GeV})^3$, $\tau_0 \sim 0.59 \times 10^6$ fm, and $\Omega^{1/3} \sim 5.7 \times 10^{10}$ fm.

The results of purely leptonic interactions of Section 12 of I are retained if (I 12.5), corresponding to (8.1), remains unchanged by the new M_W value in (9.2). This is achieved by redefining S_{F3} in (6.1) or (I 9.2a) and S_{F2} of (I 2.2) to include a factor of $2\sqrt{2}$.

APPENDIX, $\pi^+ \to \pi^0 e^+ \nu$ FROM CVC

The rate of this decay based on CVC has been given in the literature. Here, part of the exposition by Kallén (1964) will be reproduced. Notations used above which have direct correspondences to those in Kallén (1964) will be adopted below.

The decay amplitude obtained from (14.94) of Kallén (1964) is

$$\begin{aligned} S_{fi(K)} &= -i \int dX^0\ \langle \pi^0, e^+, \nu | \delta H_1 | \pi^+ \rangle \\ &= \tfrac{1}{2}\ V_M^{-1} (V_e\, V_\nu)^{-1/2} G (E_{+0}\ E_{0\mathbf{K}})^{-1/2} \{\ (K_+^\mu\ + K_0^\mu) \end{aligned}$$

$$\times \overline{u}_\nu^{(+)}(\mathbf{K}_\nu)(1 - \gamma_5)\gamma_\mu\, u_e^{(-)}(-\mathbf{K}_e)\} \int d^4X\, e^{i(K_+^\mu - K_0^\mu - K_e^\mu - K_\nu^\mu)X_\mu} \quad \text{(A1)}$$

where γ represents the usual Dirac matrices given by (A2)–(A5), (A6) of Kallén (1964) and $K^\mu = (E_\mathbf{K}, \mathbf{K})$. Further, the u's are the four-component counterparts of (8.3b) and are given by (A2)–(A24) of Kallén (1964).

The decay rate is given by (14.95) of Kallén (1964) and is written in the form

$$\frac{1}{\tau} = \Gamma(\pi^+ \to \pi^0 e^+ \nu) = \sum\nolimits_{\text{final states}} \sum\nolimits_{\text{spins}} |S_{fi(K)}|^2 / T_d$$

$$= (2\pi)^{-9} V_M\, V_e\, V_\nu \int d^3\mathbf{K}_0\, d^3\mathbf{K}_e\, d^3\mathbf{K}_\nu \sum\nolimits_{\text{spins}} |S_{fi(K)}|^2 / T_d \quad \text{(A2)}$$

where T_d denotes a long time period during which decay takes place. Introducing, as in (14.103) and (14.106) of Kallén (1964),

$$\Delta = E_{+0} - E_{00} = 4.5936 \text{ MeV} \quad \text{(A3a)}$$

$$\varepsilon = m_e^2 / \Delta^2 = 1.2375 \times 10^{-2} \quad \text{(A3b)}$$

and neglecting Δ^2 next to $(E_{+0} + E_{0\mathbf{K}})^2$, we find that (A2) has become (14.109) of Kallén (1964):

$$\frac{1}{\tau} = (G^2/192\, \pi^3)\,(1 + E_{00}/E_{+0})^3\, \Delta^5$$

$$\times \left[\frac{2}{5}\sqrt{1-\varepsilon}\,(2 - 9\varepsilon - 8\varepsilon^2) + 6\varepsilon^2 \log \frac{1 + \sqrt{1-\varepsilon}}{\sqrt{\varepsilon}}\right] \quad \text{(A4)}$$

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